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B.Sc. II Physics Hons

Paper - 4.

Boundary conditions:
Refraction of electrostatic lines of force

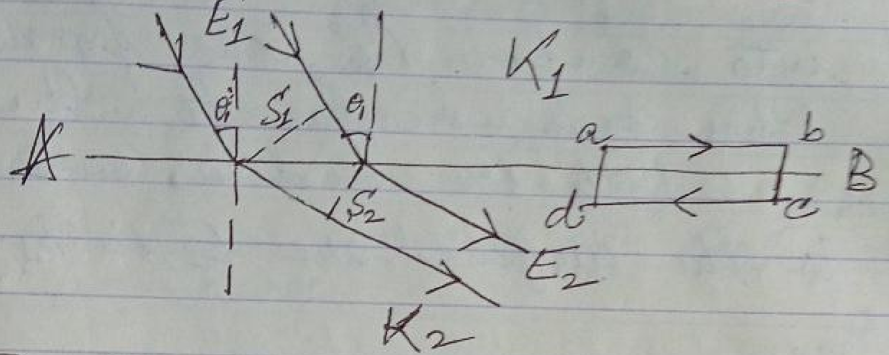


Fig. ② Refraction of tubes of force.

In many cases the tubes of force, responsible for producing the electric intensity, pass from one dielectric to another. To find the behaviour of the tubes under this condition, let us find the conditions at the boundary or surface of separation of the two dielectrics.

Let fig. ① represent a section of a tube of electric induction passing from one isotropic dielectric to another of dielectric constants K_1 and K_2, where AB is the surface of separation of K_1 and K_2. Let it make an angle θ_1 in K_1, and θ_2 in K_2 with the normal at the boundary. The electric intensity within this tube is everywhere parallel to the tube.

Let E_1 and E_2 be the electric intensities in K_1 and K_2 respectively. Since E is a vector quantity, let each be resolved into components, one parallel and the other perpendicular to the surface, in both the media. Components parallel to the surface are $E_1 \sin\theta_1$ and $E_2 \sin\theta_2$.

Now let a small +ive charge dq be placed at a point very close to the boundary. Due to the tangential component of intensity, it will move along ab. Hence work done is

$$= dq \times E_1 \sin\theta_1 \times ab.$$

Next let the charge move from b to c through an infinitesimal distance perpendicular to the surface. In this case the work done is negligible. Let the charge be now moved from c to d against $E_2 \sin\theta_2$. Hence work done $= dq \times E_2 \sin\theta_2 \times cd$. Next move the charge from d to a, the work done is again negligible. Hence by the law of conservation of energy, the total work done in

Taking the charge round abcd = 0

$$\therefore E_1 \sin\theta_1 = E_2 \sin\theta_2 \quad \longrightarrow (3)$$

(a) Thus the components of electric intensity parallel to the boundary are the same on the two sides of the boundary surface. In this condition the tangential component is said to be continuous across the boundary.

Next:

Let S' = area of the boundary cut by the tube.

S_1 = area of cross-section of the tube in the upper medium.

S_2 = area of cross-section of the tube in the lower medium.

Since the Total ~~Number~~ Normal Electrical Induction (T.N.E.I.) (or Total Normal Displacement or Flux) over the surface of a tube is constant,

$$\therefore \epsilon_0 K_1 E_1 S_1 = \epsilon_0 K_2 E_2 S_2$$

But $S_1 = S'\cos\theta_1$, and $S_2 = S'\cos\theta_2$

$$\therefore K_1 E_1 S' \cos\theta_1 = K_2 E_2 S' \cos\theta_2$$

or, $K_1 E_1 \cos\theta_1 = K_2 E_2 \cos\theta_2 \longrightarrow (4)$

or, $D_1 \cos\theta_1 = D_2 \cos\theta_2$; where D is the electric displacement or flux density.

(b) Hence the components of electric intensity (and displacement) normal to the surface are the same in both media.

Thus the normal displacement is continuous across the boundary.

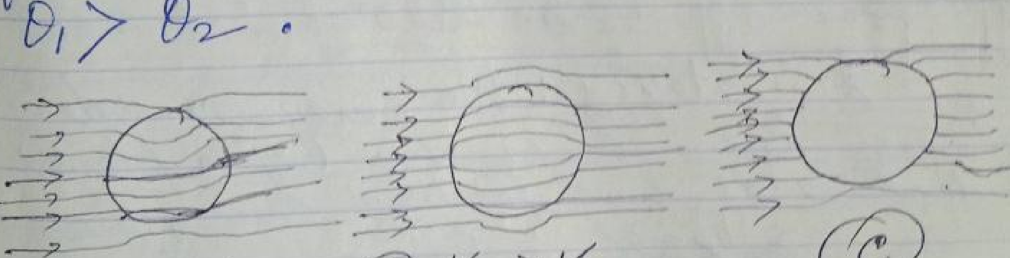
But $E_1 \sin\theta_1 = E_2 \sin\theta_2$

$$\therefore \frac{\tan\theta_1}{K_1} = \frac{\tan\theta_2}{K_2}$$

$$\text{or,} \quad \frac{K_1}{K_2} = \frac{\tan\theta_1}{\tan\theta_2} \longrightarrow (5)$$

Equation (5) enables us to find the direction of the tubes of force in the second medium, if their direction in the first medium, and K_1 and K_2 are known. In other words, Eqn. (5) gives us the law of refraction of the tubes of force, and, therefore, of electric intensities in the neighbourhood of a point on the sides of a boundary surface between two dielectrics, i.e., the law of refraction of the electric field.

If $K_1 = K_2$, there is no refraction or bending. Also if $\theta_1 = \theta_2 = 90^\circ$, even then there is no bending. If $K_1 < K_2$, $\theta_1 < \theta_2$ and, if $K_1 > K_2$, $\theta_1 > \theta_2$.



(a) $K_1 < K_2$ (b) $K_1 > K_2$ (c)

Fig. (3). K_1 is the Dielectric Constant of the sphere and K_2 that of the medium in which it is immersed.

The direction of the lines (and tubes) of force under the last two conditions are shown in Fig. 3(a) and (b); and the direction of the lines (and tubes) of force when an uncharged conductor is placed in a uniform electric field is shown in fig. 3(c) for comparision.

[For the boundary conditions in magnetism the procedure is the same and (3), (4) & (5) hold equally for magnetic intensities.

Thus; $$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2} \quad] \longrightarrow (6)$$

Force at the bound